Finding the “right” level of abstraction for patterns

Peter Baumgartner & Christian Kohls
Version 0.9 / October 5th 2013

Abstract

Patterns are abstractions of multiple instances to a core invariant structure of the solution. But what level of abstractions to choose? As each abstraction means a loss of information, we need to think about which abstractions are justified to have patterns that are flexible, open and instructive. We will discuss different types of abstractions and point out what has to be observed in this abstractions process. This will help us to build patterns that are more suitable in their practical usage and to generate a pattern language, which is more consistent and completed.

1 Why is abstraction important for pattern design?

Patterns generalize over multiple cases and capture the essence of similar structures at a mid-level of abstraction (Gabriel 1996). But “mid-level” is a wide range. Rising (2007) points out there might not be one right level of abstraction. The challenge is that patterns can be too vague or contain too many details (Buschmann 2007). Too detailed descriptions are hard to transfer to new situations. Unless stated explicitly it is not clear which structural qualities are required by the specific context and which parts of the solution structure are more general and qualify for generalization. Too abstract pattern descriptions are hard to grasp, difficult to understand and less instructive. As patterns are only general for the identified context, they are usually not universal (Lea 1994). A pattern becomes more universal when it can be used in different contexts and its solution offers different options for its implementation.

A pattern that is too abstract might become meaningless because the relevant parts are missing. What is relevant depends on the context and the experience of the person who applies the pattern. A general context implies that the solution is also stated in more general terms. The reason is that there will be many differences in the particular contexts and therefore the solution must be flexible to adapt to the particular forces found in the situation. Patterns should provide enough detail but be general enough at the same time (Coplien 1996). Patterns are mid-level abstractions and not blueprints or exact step-by-step recipes: “much of the power of patterns stems from the fact that they do not prescribe a particular implementation” (Buschmann 2007, 76).

They are rather sketches or loose diagrams that illustrate the structural quality of the pattern, i.e. the fundamental relations of the elements of a pattern. A pattern does not specify every detail but its gestalt needs to sustain: “It means, of course, that I want to make a simple picture of it, which lets me grasp it as a whole. And it means, too, that as far as possible, I want to paint this simple picture out of as few elements as possible. The fewer elements there are, the richer the relationships between them, and the more of the picture lies in the ‘structure’ of these relationships” (Alexander 1979, 81).

For educational reasons we will illustrate our argumentation with two lines of examples from different domains: One line is drawn form the domain of everyday life and should be understandable without special knowledge. In this case we have chosen as the object of attention the commonly known concept of “car”. The other line of illustration comes from our special domain pedagogical/educational environments.

First line of illustration – common sense example car: no. 1

> **Too abstract**: If we want someone to teach how to build a “vehicle” we will be confronted with intractable problems. The concept “vehicle” is too abstract because it can mean very different things such as bike, car, train or an airplane. All of these objects belong to the category of “vehicle” but are so different to each other that we need to cover an unsurmountable broad range of knowledge. The concept of “vehicle” is therefore not constitutive for draftsmen or designing engineers.
>
> **Too concrete**: On the other hand we could choose to teach how to construct a specific car model. In this case we can describe the building process in all its details. A specific model of a car, however, is no longer a design pattern. Being fully specified there is no more variability. It is only a template and each instance created by that template looks similar except for surface properties such as colour.

Another failure of too concrete descriptions is to start with specific parts rather than describing a larger whole. For example, we could start with teaching how to construct an auto body. There are many different types of auto bodies and indeed there exist body shops and specialists for bodywork, called coachbuilder in Britain. But even if there (should) exist a pattern language for coachbuilders it seems not the right way to *start* teaching with the subject of body construction. Bodywork is too specialized and a carriage or auto body is not a self-contained whole. It always needs the reference to the object it will form a part, the object to which it belongs.

The middle level: However, if we use for our educational focus the abstract concept of a "car" it is clearer what is meant. Yet there are actually millions of ways to build a car. The pattern "car" is generative, real forms can be derived from that concept. The more specific patterns "cabriolet", "SUV", "van" still are generative for there are many variations of these subclasses of cars. They are still a challenge to develop and to construct and are therefore constitutive for car designers or generally speaking for designing engineers.

Second line of illustration – specialist knowledge example teaching: no. 1

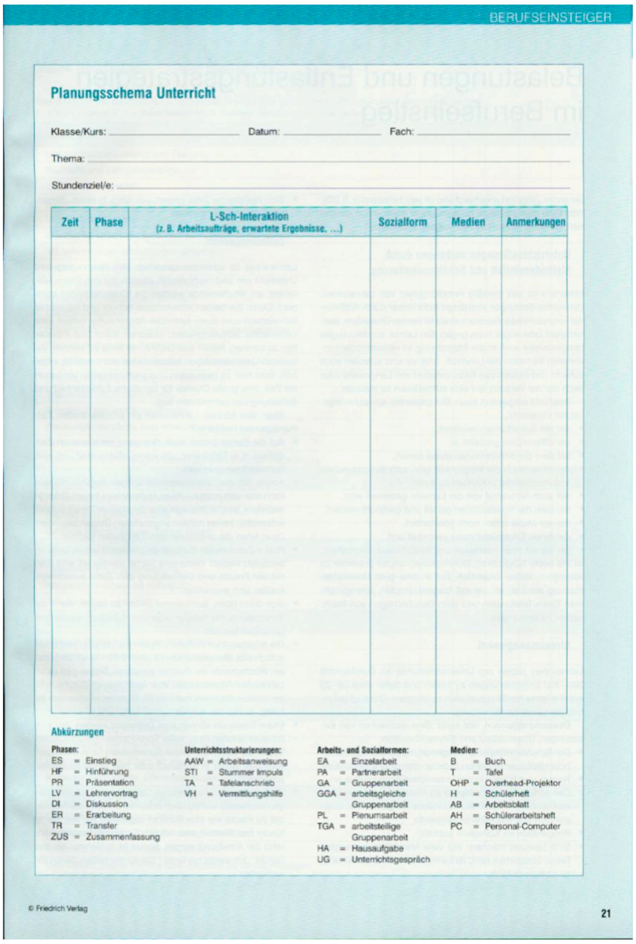
Too abstract: Experiential education or even experiential learning is certainly a very interesting research field in its own. But is it also a suitable concept to teach apprentice-teacher how to prepare for their classroom teaching? We believe not, it is too abstract to specify a teaching design. There are not only many strategies for experiential learning but quite different approaches as well such as visit, excursion, exploration, hands-on training, internship, project, legitimate peripheral participation (Lave and Wenger 1991) just to mention a few of them (cf. Baumgartner 2011, 267).

Too concrete: But if we try to explain apprentice-teachers what to do every couple of minutes and how to sequence these episodes then we are surely too much into the details. Such explicit plans mostly do not work because nobody can foresee the problems we confront in real life as we cannot dismiss the "thrownness" in our Being-in-the-world as Heidegger put it (2008, 174). Unexpected "interruptions" like a failing technical (teaching) device, a (complicated) question by a student are able to overthrow every detailed planning. But even worse: To teach apprentice-teacher on such low level of abstraction prevents to get down to the nitty-gritty of lesson planning. Unfortunately such approach is still very common as one can see of some "tools" apprentice-teacher are forced to use for their lesson planning (cf. Figure 1 and 2 next page). At this concrete level teacher students are forced to plan their interactions as micro-interventions in the range of some minutes and they have to think ahead what kind of medium and social form they will apply to their micro-teaching episodes (Baumgartner 2006).

The middle level: Instead of teaching superfluous in-depth details of lesson planning we should concentrate our teaching to teacher-novices how to plan an excursion, an exploration, a project etc. These mentioned approaches aim at some kind of immersion into reality to get his/her own life experiences and are in contrast to many other families of teaching patterns like teacher-centered teaching, problem-based learning, learning through case studies etc. Even if all these teaching models belong to a family of teaching patterns we could call "experiential learning" they are quite distinct in their structure and procedures. They are open for much variation. Two projects for instance are never identical, even if they address the same problems and aim at the same goals (Baumgartner and Payr 1997).

Datum:	Stunden-Nr.:	Unterrichtsgegenstand:	Klasse/Jahrgang:
Stundenthema:		Unterrichtsziel:	
Eingangsvoraussetzungen:		Querverbindungen zu anderen UG:	
Aufbau der Unterrichtsstunde		**methodisch-didaktische Anmerkungen** (z.B. Medien, Sozialform, Arbeitsmittel, Übungsnummern, Arbeitsanweisungen etc.)	Dauer
1	Stundenbeginn – Einstieg		max. 10'
1.0	Eintragung ins Stundenbuch		
1.1	Hausübungskontrolle		
1.2	Wiederholung (Mitarbeitsbeobachtungen)		
1.3	Kontrolle der Eingangsvoraussetzungen		
1.4	Querverbindungen zu anderen UG		
1.5	Überleitung zum Stundenthema		
2	Erreichung des Unterrichtsziels: Lehr- und Lernschritte		
2.1			
2.2			
3	Stundenende – Ausklang		max. 10'
3.1	Stundenzusammenfassung Kontrolle: Unterrichtsziel erreicht?		
3.2	Hausübung: bis:		

Figure 1: Form for Lesson Structuring (Becker et al. 2007, Friedrich Jahresheft XXV:64)



BERUFSEINSTEIGER

Planungsschema Unterricht

Klasse/Kurs: ____________ Datum: ____________ Fach: ____________

Thema: ____________

Stundenziel/e: ____________

Zeit	Phase	L-Sch-Interaktion (z. B. Arbeitsaufträge, erwartete Ergebnisse, ...)	Sozialform	Medien	Anmerkungen

Abkürzungen

Phasen:
ES = Einstieg
HF = Hinführung
PR = Präsentation
LV = Lehrervortrag
DI = Diskussion
ER = Erarbeitung
TR = Transfer
ZUS = Zusammenfassung

Unterrichtsstrukturierungen:
AAW = Arbeitsanweisung
STI = Stummer Impuls
TA = Tafelanschrieb
VH = Vermittlungshilfe

Arbeits- und Sozialformen:
EA = Einzelarbeit
PA = Partnerarbeit
GA = Gruppenarbeit
GGA = arbeitsgleiche Gruppenarbeit
PL = Plenumsarbeit
TGA = arbeitsteilige Gruppenarbeit
HA = Hausaufgabe
UG = Unterrichtsgespräch

Medien:
B = Buch
T = Tafel
OHP = Overhead-Projektor
H = Schülerheft
AB = Arbeitsblatt
AH = Schülerarbeitsheft
PC = Personal-Computer

© Friedrich Verlag

21

Figure 2: Form for Lesson Planning (Böhmann and Klaffke 2010, 21)

2 What are the main characteristics of abstractions?

2.1 Different levels of reality

Our examples show: There are different levels of abstractions in which we conceive reality. Some philosopher like Nicolai Hartmann (1964) and Michael Polanyi (1969; 1974; 2009) claim that this is not only a perception or construction of our mind but that reality itself consists of different levels. This is a general principle of our world we have to come to grip with. The mentioned philosophers have investigated the laws that control the levelled structure of reality. In this paper we cannot discuss all the details but need to outline at least two aspects of their findings.

2.1.1 Inclusive hierarchy

The levels are not just layers ordered one on top of the other, but they form an inclusive hierarchy where the "higher" ones include all the "lower" ones.

First line of illustration – common sense example car: no. 2

The car includes among other things such as motor, chassis, car tires, seats and a steering wheel also our previously mentioned auto body. All these parts could be conceived as members of a "lower" level of the reality of a car. Screws, bolts and fan belt are instances of a layer that is even lower. All objects of a (relatively) lower level are included in the higher level; they form together an inclusive hierarchy with different levels. All these items are tacitly included when we are referring to a "car". In contrast: a driver, a street or for my sake a brick are not parts of a car. These objects belong to different domains.

Second line of illustration – specialist knowledge example teaching: no. 2

Some educational researcher like Flechsig (1983; 1996) and Baumgartner (2011) have also conceived the educational domain as consisting of different levels (Cf. Figure 3 next page, showing the inclusion principle by using an "onion"-metaphor instead a pyramid-metaphor). The importance of this conception lies in the claim that different levels are subject to different laws. In order to design educational situations we have to investigate the underlying principles and take them into account.

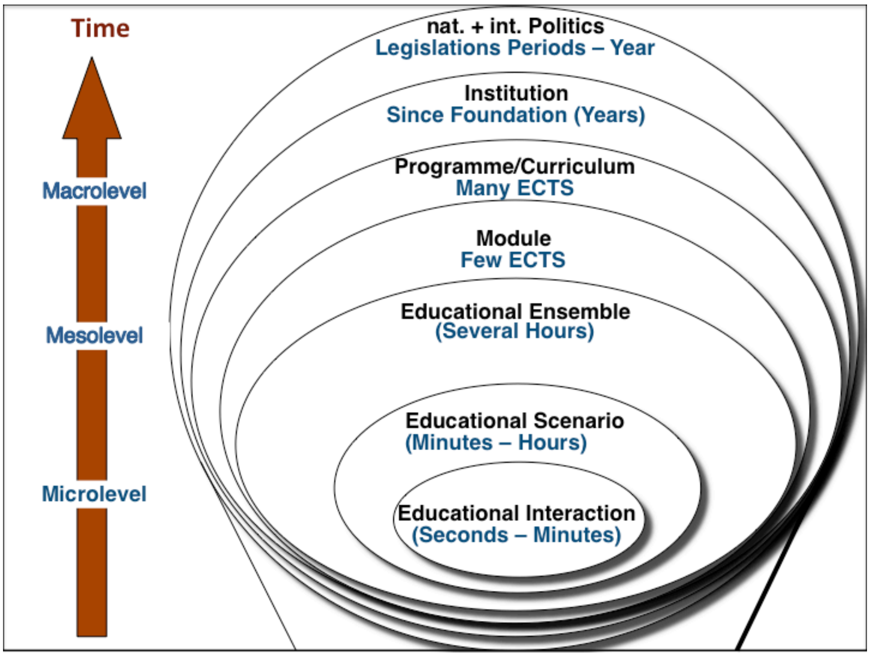


Figure 4: Inclusive hierarchy of different levels of educational design (Baumgartner 2011, 66)

2.1.2 Emergence

Another important issue is the fact that characteristics of "higher" levels cannot be reduced or explained from the laws or principles of their parts from "lower" levels. The property "liquid" is not present in one molecule H_20

but is an emergent property of the higher level “water” caused by the *organisation and interaction* of parts to each other from the lower level.

First line of illustration – common sense example car: no. 3

It is obvious and trivial to say that the mode of operation of a car is not explained by the additive combination of its part. It is the specific organisation and interaction of all the parts that cause the car to work.

Second line of illustration – specialist knowledge example teaching: no. 3

It is not so easy to give examples of emergence in the humanities or in the social sciences like in physics or chemistry. But intuitively we do know that of the level of curricula emerge other properties that could not be explained solely from the standpoint of their modules. To investigate these new characteristics and their interdependencies between the different levels is one important lesson we have learned from the level-of-reality point of view.

It seems to us that the level of Educational Scenarios is the most important one in preparing for teaching classes. We give the reason for this opinion in section 3. But anyway: Always connected with a certain level of abstraction, there is a specific set of questions we need to answer in order to design appropriate interactions: What kind of educational interactions (= lower level) do we need to form a suitable educational scenario? What kind of educational scenarios should we use and how should we orchestrate them into the classroom in order to shape the subject-matter block (= higher level) more effectively for the student learning experience?

In this conception the chosen (“right”) level of abstraction turns out as a middle strata because we have to switch out attention between lower and higher level in order to figure out the necessary building blocks (= lower level) and to estimate the emerging consequences (= higher level).

2.2. Granularity and scope

In choosing the appropriate level of reality for pattern construction we have to account of two other features in the inclusive hierarchy of abstractions. One is the distance between different objects at the same level of abstraction and the other one is the distance of the different levels themselves. We will call the first one internal granularity and the second one external granularity.

Internal granularity measures to what extent a certain level of abstraction is populated with objects or processes. The more items we account for the abstraction level in question the higher is the chance that two items are quite similar and the less far away are they in our internal graded cognitive representation.

External granularity on the other hand is a measure how far away two levels of abstractions are situated. Granularity can be compared with the artificial composition of a photo: internal granularity looks at the chosen level of pixel resolution, external granularity focuses at the texture, the overall structure of the picture.

The same objects or processes in different combinations of included hierarchies can be a separate hierarchy (addressing external granularity) or just another family member in the same hierarchy level (addressing internal granularity). It depends on the overall structure of the hierarchy.

Scope is another important technical term in our argumentation. Scope indicates the share of influence to change reality accounted for by a certain abstraction level. Scope determines the relative position in the inclusive hierarchy of abstractions. Scope is dependent from the focus of our interest and the target group we plan to address.

Scope refers therefore to the realm of reality and the circumference covered by the level of abstraction in question. Designing patterns of lower levels of abstractions are linked to seemingly smaller changes if we just look at the objects or processes themselves. But as a new design of these smaller items could also change their interaction with other objects or processes in the world they could have (through emergence) profound impact of higher levels of reality. For instance the appearance of small gadgets like the smartphone has changed significantly our communication habits and life style.

Coherence: The most important thing with granularity is not the absolute size of the distance of their items but their consistency throughout the different abstraction levels. If there are big differences in the number of objects or processes populating the different levels we may ask if we have broken up the levels of reality appropriately.

The division of a larger whole into its parts has not only implications to the gap between the different levels of abstraction (external granularity) but has also consequences for internal granularity. If the overall structure is very coarse it does not make sense to go into the very details of just one of those levels.

First line of illustration – common sense example car: no. 4

External Granularity: We have a greater external granularity when we construct a hierarchy with just the levels of "vehicle← →car" then we would have if we construct "vehicle← →motor vehicle← →means of transport← →personal transport← →car". In the second example the "space" between vehicle and car is filled up with other levels of abstraction and reduces the distance or gap between different levels. It depends of our problems we want to solve and our target group we want to address what would be the more appropriate break down of reality into abstractions levels.

Internal Granularity: If we consider one of our specified abstraction levels (e.g. "car") the difference in internal granularity would be exemplified by "passenger car← →truck" versus a list of models e.g. including all brands of all car manufacturers. We could even distinguish between different years of manufacturer. As we said above this would go too much in details; the changes would focus on details that are not functional relevant like colour.

In the chain "vehicle← →motor vehicle← →means of transport← →personal transport← →car" there is no place for a separate tier of "motor" abstractions. It could be only a detail in the abstractions levels of cars. But in the structure "environment← →traffic← →vehicles← →car← →motor" does "motor" fit as separate level of abstraction in order to tackle pollution problems.

Even if we have chosen to stop the external granularity at a certain level, let's say at the level of "cars" then we still could depicture different amounts of information about the family members of this chosen level. We could for instance include or exclude details such as the motor. Again it depends on our questions and target groups: If we are going to find an appropriate traffic solution for suburban community we could exclude the different kinds of motors as irrelevant; but if we are going to improve the environmental situation the construction of motors could be important for our problem.

Scope: If we design patterns for a traffic solution each pattern would have a much greater impact (=broader scope) than we would have if we would focus just on the design of passenger cars. For the traffic solution we would need also to incorporate the population, housing, structure of streets, parking places, traffic lights, public transport etc. In the design of passenger cars all these items are of minor relevance.

Consistency: But whatever we chose and what kind of target group we are going to address we need to be coherent in our granularity. In the hierarchy "environment← →traffic← →vehicles← →car← →motor" we would not go into the details of the construction of traffic lights (level "traffic") as these concerns are not important for environmental issues. But traffic signals are relevant in their distribution and timing sequence within the area we focus on. On the other hand it could be very relevant to go into very details of car construction, as their power engine, weight and speed are highly relevant for particulate matter and other environmental impact.

Target Group: If we are going to write patterns to educate bodywork designer we should write patterns for all parts that are functional for the visual appearance like hoods, head- and taillights, doors, grills etc. but we would not include screws and bolts. But if we would address coachbuilder then we would even need to go into the different kinds of bolts like axle bolt, distance bolt, and their different forms like S-,T- U-bolts.

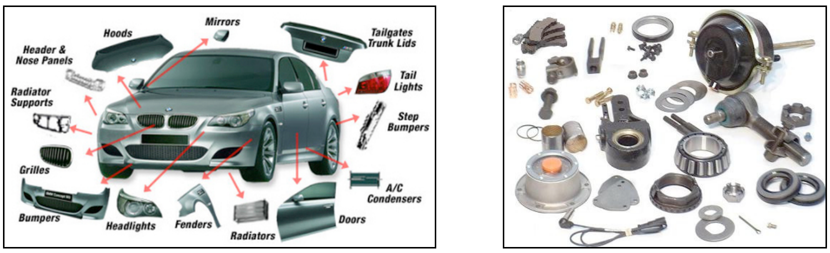


Figure 5: Different focus changes the overall abstractions hierarchy (external granularity) and how much one has to go into the details of one hierarchy (internal granularity)

Second line of illustration – specialist knowledge example teaching: no. 4

External granularity: We have already argued for a greater external granularity for training purposes of apprentice-teachers. Instead to combine the tiers of "education scenario← →education ensemble← →module" into one category may be called "didactical design" or just "teaching" or "instruction", we suggest cutting reality into more tractable (smaller) layers and breaking up the very general category of "teaching".

Internal granularity: The more items are ascribed to one level the finer graduated are the differences. Again it depends on our educational goal and the target group we want to address.

One could argue that novice-teacher should not be confronted with too many details, as they could be intimidated purely from the sheer enormity of different options. So we should constrain our instruction to novice-teacher to the prototype or some very high ranked "good" examples. On the other hand one could counter that the complexities of reality should not be concealed completely because it would make it more difficult to find answers to practical challenges. Whatever argument is valid it is clear enough that to educate expert teachers one certainly would need a more refined abstraction layer with more objects or processes e.g. a bigger family with more members.

#	Layers of Educational Actions	Levels of Educational Abstractions					
		Descrip-tions	Methods		Principles	Dimen-sions	Cate-gories
			Patterns	Models			
		1	2a	2b	3	4	5
H	Internat. Systems						
G	Nat. Systems						
F	Institutions						
E	Curricula						
D	Modules						
C	Ensembles						
B	Scenarios						
A	Interactions						

Figure 6: Educational Taxonomy (Baumgartner 2011)

One of the authors (Peter) has presented in his educational framework (Baumgartner 2011) several level of abstractions for every different level of education action. This demonstrates that there always could different level of abstractions to describe the same educational situation.

We have already described the levels of the y-Axis as an inclusive hierarchy (cf. figure 4), so we can concentrate on the x-Axis of figure 6: The hierarchy of abstractions levels consists of five layers: (educational) Categories← → (educational) Dimensions← →(educational) Principles← →(educational) Methods← →(informal educational practice) Descriptions. Each of these different abstractions levels has a different scope and is addressed to different design problems and targets groups. Educational categories form the most general abstraction level and address the design of educational theories. On the other pole we have informal educational descriptions lacking abstractions at all but are suitable for personal day-to-day communication. One essential line of argument in this hierarchy of abstractions is the claim that the formal format of education patterns are better suited to transfer practical know-how than the more abstract and stale format of teaching models.

Focusing of the action layer of educational scenarios Peter has worked out 21 binary relationships resulting from 7 basic categories 26 educational dimensions, educational principles (130) educational models (133, by far not complete, only a subset of possible teaching models) and therefore subdivided and grouped together into (so far) 18 families of educational models. These magnitudes of family members for each abstraction level are within our proposed limits (20 to 230 resp. 150) as we have outlined later in this paper.

Scope: It is obvious that the design of educational policy by the government has a much broader range of impacts as the design of micro-didactical interventions by a teacher. If we want to cover such a broad field, we need to cope with the complexity on a higher level of abstraction. In the scope of educational policy it is appropriate to abstract to experiential learning and only consider the emergent impacts in the context of the whole policy. However, if we want to design courses that include experiential learning we are changing our scope to that part of the larger whole. Therefore, we need to change to a granularity level that makes the differences between visit, excursion, exploration, hands-on training, internship, project etc. visible.

2.3 Different abstraction strategies?

There is a lot of research going on about then nature and characteristics of abstraction processes (Bjørner 2005; Kaschek 2004; Fine 2002). We cannot go into the details here but we will outline three important abstraction strategies:

- We can focus our attention to certain feature with has the impact of reduction. Sometimes this strategy is called abstraction by isolation, as we segregate some properties as irrelevant. We will call this process "abstraction by segregation"
- We can lump different elements in one category by focussing on structural similarity. While the details of the considered criteria may vary their relations are kept intact. Sometimes this strategy is called generalisation, which is in our opinion a misnomer as all kinds of abstraction results into generalization. We will therefor call this abstraction procedure as "abstraction by aggregation".
- If we abandon the somewhat atomistic and static view of individual elements we can look of the dynamic interaction of different elements and focus of the emergent qualities. Different combination of interactions results into different emergent properties. We call this procedure "abstraction by emergence".

For the sake of simplicity let us consider three very primitive systems of points that are distributed along a one dimensional line. While these three systems have been created artificially for illustrations they are real systems. We choose such a primitive system of points for a good reason: we can start at the most concrete level that captures the real properties of the points without any abstraction in the beginning.

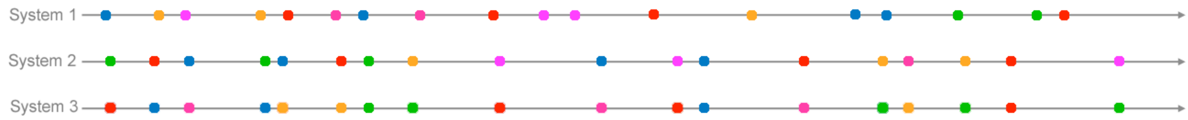


Figure 7: Systems of points

As you can see, the points of our three systems have different colours. That means, each point has at least two properties really: the **position** on the line and a **colour**.

2.3.1 Abstraction by segregation

A feature of the actual object is omitted. Positive abstraction means according to Erdmann (1892) to isolate the relevant features and focus on them; negative abstraction isolates irrelevant variations between several objects. As we are distinguishing different kinds of abstraction we will use for these kinds of abstractions the notions of positive and negative segregation

Abstraction by segregation is the most intuitive abstraction procedure. Features that are omitted in an abstraction will have no correspondence in the resulting model. Features can be omitted if they are not relevant or it is clear how the appropriate option is selected, i.e. how the best alternative is chosen. Because we omit only irrelevant or superficial features, this form of abstraction should not cause any changes to the granularity or scope. The purpose is to focus on those features that are relevant for a specific question or task. A common misuse of this form of abstraction is to omit features that are essential differentiators. For example, if we want to focus on the nutritional facts of an apple we could abstract from its actual taste. However, if we want to prepare a delicious meal we should not abstract from the actual taste. In this case treating an apple and a lemon as equal could be fatal.

For our system of points let us assume that we are only interested in the positions of the points, not their colours. This would be an example for "abstraction by segregation". There could be two reasons why we focus on the points only.

(a) We consider the colour as irrelevant. That would be negative segregation, taking everything away that has been identified as irrelevant for specific question.
(b) We consider only the points as relevant. That would be positive segregation, focussing only on the properties that are relevant for a specific question.

The result of positive or negative segregation is the same. However, for more complex system the process of identifying relevant properties vs. identifying irrelevant features may differ strongly. Even for our primitive system, positive and negative segregations differ in their justification. For positive segregation we argue why one property is considered relevant. For negative segregation we argue why one property is considered irrelevant.

For our primitive system we can argue that we want to focus on spatial relations of points and therefore only their positions are relevant.

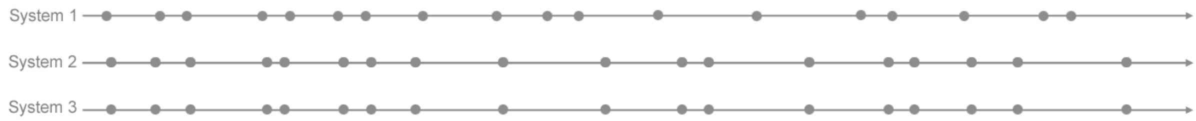


Figure 8: Abstraction to positions

This abstraction did no harm to our dimension of interest: abstracting from the actual colour does not have any effect on the spatial positions. The properties are independent and isolating one property does not change the behaviour or effect the other In spite of the abstraction the spatial distribution of points is absolutely equal to the original system.

If we consider the abstracted systems 2 and 3 we will find that both are now identical because of the abstraction. We can treat system 2 and system 3 equally as far as the spatial relations are concerned. Instead of three different systems, we only have two different systems to cope with because 2 and 3 are now identical. The dimension of interest remains untouched.

First line of illustration – common sense example car: no. 5

Each car has an abundance of properties: body colour, maximum speed, production date etc. Which properties matter depend on the questions one wants to be answered. If we are interested in the amount of charge a car can transport, we can abstract from its colour and production date. If we are interested in its aesthetical features we must not abstract from the colour but can abstract from its speed and its transportation properties.

Second line of illustration – specialist knowledge example teaching: no. 5

Each teaching situation is a complex setup with many properties: the feelings and motivations of students and teachers, the teaching location, the group size etc. However, to describe a teaching situation appropriately we are usually not interested in the height and weight of the students, their names or hair colours.

2.3.2 Abstraction by aggregation

A feature of the actual object is covered but not precisely. While abstraction by segregation omits irrelevant features, abstraction by aggregation omits irrelevant variations but should preserve the general structure (Wundt 1907). For example, the different widths of a street are mapped to one line thickness.

Let us consider this abstraction for our system of points. Remember that each point on the line is a real element (although being in an artificial system). In the process of abstraction by aggregation, individual element can be put with one another into the same category. Abstraction in this case means that we will consider two elements as equal if their differences are within a tolerable extend. That is on the higher levels we do not differentiate between the actual differentiations of the elements.

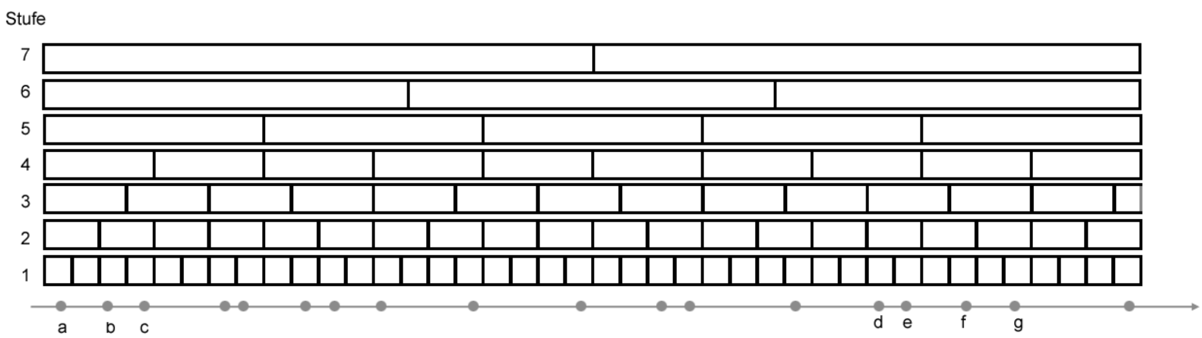


Figure 9: Abstraction by aggregating different values

Consider figure 9. At the chosen granularity, at the first level of abstraction each point is mapped to a different category – a clearly distinguished section. While we loose the exact position of the point at this level of abstraction, all elements that are differentiated in reality are still represented as different. This differentiation, however, gets lost on higher levels of abstraction. For example let us consider the points a, b and c. At the 1st level of abstraction these points remain distinguishable. At the 2nd level of abstraction b and c fall into the same category. At the 3rd level there is no more differentiation between a, b and c. Because of the broader internal granularity they fall into the same category.

In an abstract representation each point could be represented by its average value or its range of values. What ever the choice is, the abstract representation no longer differentiates between the actual elements. Likewise, the differentiation between points d and e would already get lost on the first level of abstraction. Therefore we have to be aware that the abstraction process by aggregation will not destroy the structural information we are interested in.

Abstraction by aggregation that do not differentiate between the actual elements are not always a bad thing. For example, even on the higher levels a and b are clearly distinct from f and g. Instead of differentiating between cat and dog, we can categorize them as pets. This means loss of information but we can still distinguish pets from wild animals, or mammals from reptiles.

Abstraction by aggregation may or may not lead to a loss of structural information if we consider the interplay of several elements of the system. We can consider the points a, b and c as a sub-system. On the 1st level of abstraction the differentiation of its elements remains intact and the structure of relations is preserved. On the 2nd level of abstraction, however, b and c fall into the same category. Hence, the structure of the aggregated system is different from the structure of the real system. Emergent behaviour between the elements may no longer be captured. The interplay of two “pets” is different from the interplay of a “dog” and a “cat”.

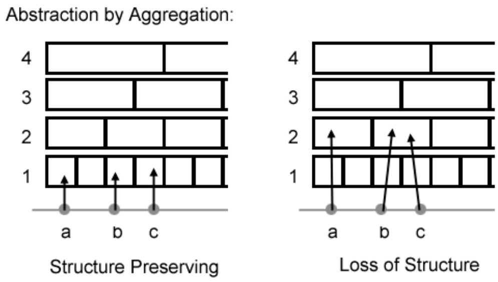


Figure 10: Abstration by aggregation with and without loss of structure

Patterns must preserve the structural relations we are interested in. We generalize over the detailed values of elements but the elements and their relations remain distinct and intact. Abstraction by aggregation should be done to a maximal extent that is before the structure gets lost. Then we simplify over multiple cases to one general structure and show a range of potential configurations rather than specific ones. The structural relations we are interested in are preserved and so are the emergent qualities arisen from the lower levels. Another approach to abstraction is to focus on the emergent qualities only rather than the actual structure, as we will see in the next section.

First line of illustration – common sense example car: no. 6

We can abstract from the actual sizes of wheels, the exact lengths and width of the body etc. However, the structural relations between the parts must remain intact. Moreover, the actual values need to be within a range of valid values. At a certain length of a body, we would perceive a limousine or a bus rather than a car.

Second line of illustration – specialist knowledge example teaching: no. 6

We can abstract from the actual number of participants, the actual duration of learning sessions or the length of an introduction if these values are within reasonable limits. For instance it may be not relevant if we organize a learning experience for 10 or 13 participants, if this experience lasts for 45 or 55 minutes and the structural relations of activities remain intact (e.g. 1 teacher makes the introduction,in front of a group of learner).

2.3.3 Abstraction by emergence

Abstraction to emergent qualities is discussed in the context of both information and complexity theory (for example: Gleick 2011; Mitchell 2009; Holland 2000) but the idea of complex abstractions (i.e. the complex grouping of features to a whole) has already been discussed by St. Thomas (cf. Bobik 1963). Emergence means that the complex interplay of elements of a lower level leads to new effects or laws on a higher level. However, we do not need to consider the specific configuration of the components as long as the quality on the higher level is preserved. For example, to understand how a car works we do not need to analyse its chemical composition (Von Baeyer, 2004).

In our system of points we have already observed that the abstraction of multiple elements as one intact system has implications for the right level of abstraction. While we required that both the structure and its emergent qualities remain intact, an abstraction to emergent qualities considers the actual micro-structure on a lower level as a “black box” and abstracts to the behaviour in a macro-structure on a higher level. We no longer consider the details of the actual parts, but the emerging whole. As a consequence, instead of considering multiple elements on a micro-level, we only consider one element on the macro-level.

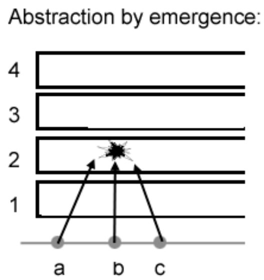


Figure 11: Abstraction by emergence

Abstraction by emergence means to see different objects as one whole. As a consequence we introduce a new level of reality, or a new dimension. If we have three points on a line each point can be specified by one value. However, there are implicitly new qualities: the distance between the points may be equal, or two points might be very close whereas the third point is far away. In two-dimensional space, such qualitative relations are easier to see: if four points are arranged in the right way they can form a square. However, the “squareness” (the property to be a square) is not due to a single point but the result of the emergent interplay of all four points. While on the level of points, we have four elements that can be differentiated (four different points), a square is a single element of a higher level. When we speak of squares we do not need to specify the single points anymore (until we want to actually draw a square).

First line of illustration – common sense example car: no. 7

The single parts of a car are one compound concept on a higher level of reality. When we abstract to emergent qualities we change the dimension. Very different forms can have the same effects, i.e. be located closely to each other in the new dimension: different vehicles can transport us to a destination. On the lower level of reality objects may be very different whereas on the higher level different objects may have the same effect. The many different possible levels of fuel in a car (many micro states) are mapped to enough/need to refill (two macro-states). At the level of the macro-state, it does not matter which actual micro state there is. To know that the tank is going to be empty is enough to know that we should look for a gas station. Easily we could imagine for practical reasons on the higher level of behaviour a third state called: “Emergency!” This state communicates, “Watch out, you are really low on gas. Immediately look for a gas station!” – This examples shows that number and type of macro-states are controlled by the desired (emergent) behaviour on the higher level.

Second line of illustration – specialist knowledge example teaching: no. 7

The actual number of learners is mapped to different kinds of group sizes (small/medium/large e.g. three macro-states). Peter in his “taxonomy” has distinguished 5 different macro-states: individual – couple (2 members) – small group (3-7 members) – middle (8-30 members = class room) – big (30+).

This illustrates that types and numbers of macro-states are influenced by (theoretical or empirical) assumptions about emergent features coming up in the higher abstraction level. With the relevant new events of MOOCs it is questionable if the 5 macro-states mentioned above are sufficient and if there wouldn’t be other macro-states besides 30+ group members.

> Or more generally speaking: Types and numbers of macro-states are a function by the possible emergent qualities of the macro level. In the “taxonomy” *all* 26 educational dimensions are mapped equally to 5 macro-states (educational principles). From the point of the new consideration in this paper this kind of mapping for symmetrical reasons is not correct anymore and should be revised to incorporate empirical or hypothesized emergent educational qualities on the higher level.

As we have learned by the process “abstraction by emergence” one major problem of abstraction is often that the elements are tackled independently. However, decoupled objects are only nearly-independent. They still rely on the context. That is why pattern descriptions should take care of explicating the dependencies to other parts of a larger whole.

The very common and often used two abstractions procedures (“abstraction by segregation” and “abstraction by aggregation”) do not only miss emergent behaviour but they are also problematic for static mapping.

2.3.4 Structure preserving abstractions

When we are looking for patterns it is important to understand that abstractions by segregation, aggregation and emergence should be done only when the structure is preserved or can be re-constructed. In these cases, the relevant information is still there. In the case of abstraction by segregation there should be no features left out that are essential to the structure we are interested in. Abstraction by aggregation means that similar micro-structures are identified and abstracted to one modelled structure. These similar micro-structures can also be abstracted to a macro-structure on a higher level, i.e. to an emergent property. If a macro-structure is clearly related to similar microstructures, then we can easily derive the actual structure by knowing the macro-structure only. The important information for its implementation is available at least by reference. However, an abstraction to emergent qualities does not always reveal the information that leads to specific structures on the lower level because different interactions in the microstructures can lead to the same effects on a macro-level. Abstraction to emergent properties means that parts of a solution can be implemented in very different ways and no information is provided about the actual structure. This requires that a user of such an abstract pattern knows how to unfold it into a more specific one. A designer needs to know how to get from a more abstract pattern to a more concrete pattern. In order to do so s/he must know the patterns of lower abstraction levels and how to compose those to larger patterns. Such knowledge is captured in pattern languages.

If we abstract forms to their emergent effects they need to be embedded in a larger scope. By dividing a large structure into smaller parts we reduce the complexity, provide choice between alternatives, and can represent each part on different levels of abstraction. Essentially this reduction of complexity is the reverse procedure as we have already covered under “abstractions by segregation” and could be called accordingly “concretization by segregation”. What strategy to write a pattern language is preferable? Should we choose the top-down or the bottom-up strategy? We recommend starting with the bottom-up strategy. Why? In the top-down procedure there is a trap pattern writers have to be aware: if we just think on the reverse procedure of “abstractions by segregation” we may not limit our pattern language to just one concretization for the mapping on the lower level. This would mean that we do not leave possibilities for alternative design on the micro level.

The choice between alternatives is a sign of openness. Openness is important in order to take the specifics of an actual situation into account. Openness refers to the agility and adaptability of patterns as well as their compatibility with other patterns (Lea 1994). In order to adapt to the specifics of a situation, a pattern is more open if it allows a variety of sub-patterns and connects to various other patterns, i.e. it can be used in the context of alternative patterns and its resulting context offers choice between patterns.

Yet if we let too many parts of the pattern’s structure unspecified we are loosing its gestalt. That means we can no longer “see” its form as a self-contained whole. We therefore have to find a compromise where a certain amount of openness is still provided but not too much in order not to loose the gestalt

If we proceed from the lower level to the upper level we have learned to collect and investigate all different possible solution (= “pattern mining”). Sure, to focus on the lower level and to abstract from there is a process more complicated but it prevents us from limiting our thoughts to just one “best solution”. But the best way would to use both procedures, starting from the bottom-up strategy – as we mentioned above – immediately followed by the top-down perspectives and continuing to use these two approaches alternately in a iterative loop.

3 How to choose the most appropriate level of abstraction?

If there are different levels of reality, which we frame cognitively as abstractions, so the question arises: How can we detect the most appropriate level we should focus in order to solve a certain problem? It is obvious: Some of these levels are more adequate for our design tasks than others. Maybe there is just only one level suitable for our special problem we are going to solve? Certainly the subject area where our problems are situated is important for our decision as well as the target group to which our teaching patterns are directed. But this is trivial and is not enough to specify the most suitable level of reality we should focus on abstractions.

We believe that the main criteria to choose a proper level of abstraction are embedded in real life encounters. Both lines of exemplifications show that we need to confront a real life object or process. Neither "vehicle" nor "experiential learning" qualifies in this respect. But to be embedded in real life is not enough. The object or process has to be a self-contained whole, which is independent and has – so to speak – a life of its own. Neither "auto body" nor "micro-intervention" qualifies to this restriction.

3.1 Abstraction and models

Whatever abstraction principle is at work, an abstraction is a mapping from the actual world to a model. Such mapping is a function, f: $X \rightarrow Y$, where the value of X is mapped to another value Y. Maps in general are models that correspond to some other structure. If $X = \{x_1, x_2, \ldots, x_n\}$is a list of details to be modelled, then $Y=\{ y_1, y_2, \ldots, y_n \}$ could be a list of corresponding features in the map (Holland 2000, 30).

However, as the saying goes, the map is not the territory. A map does not only transform the original structure to corresponding features (e.g. using a different scale) – it also skips many details and alters the phenomena: "[abstraction] does not accept the phenomena as they are, it changes them" (Feyerabend 1999, 5). For example, a map may not show the trees, flowers or stones of the actual landscape (abstraction by segregation). Different diameters of a street are reduced to a line of constant thickness (abstraction by aggregation). Moreover, maps often use symbols to represent certain types of entities in the landscape: cities are symbolized by a dot or a small house icon represents cottages. These icons do not tell us anything about the actual shape of the city or the houses, yet we know which effects or functions these entities have (abstraction by emergence).

We said that an abstraction is a mapping from the actual world to a model. Abstraction by segregation and abstraction by aggregation – are intuitively understood because they map reality to model in a static way. But these are static abstractions because they do not account for new emergent features that may come up as a result of the interaction between the individual elements of a class of objects or processes.

- **Concrete models**: They are real, physical objects or processes intended to map or represent some generalized phenomenon. A model can even be a structure that actually exists in the real world, i.e. a model citizen, a model student or a model school. In such cases the model is just an instance of the class it models (Goodman 1976).
- **Mental models**: A model can be a mental construct that represent real, hypothetical, or imaginary situations (Johnson-Laird 1983) . In this group we will find the important group of mathematical models.
- **Computational Models**: A model can also be something that we artificially create, e.g. a simulation or a model of molecules to better grasp their structures. In that dynamic case we try to map the behaviour of a system and have therefore also to account for the impacts of interactions (Weisberg 2013).

Not only the computational model but also concrete and mental models can be used to map system behaviour. In that case segregation and aggregation of objects is not adequate because theses types of abstractions cannot account for new features, which may emerge through the interactions of their elements.

It is important to understand that mapping does not only mean to create a one-to-one correspondence between objects as we have illustrated above with "street – line" and "house icon – cottages". A model can exist on various levels of abstraction (including concrete manifestations) as long as the essential features of what is modelled are not lost. In a model we find corresponding features for all relevant information. There is an "isomorphism" that preserves the relevant information: "The word 'isomorphism' applies when two complex structures can be mapped onto each other, in such a way that to each part of one structure there is a corresponding part in the other structure, where 'corresponding' means that the two parts play similar roles in their respective structures" (Hofstadter 1979, 49).

A dynamic model preserves the quality that emerges from the complex interaction of the elements of a substructure. This *mutual dependency* is not explicit but implicit in the order of each instance (Bohm 1981). The structural quality of the form emerges from the interplay of its part and not from the single positions of the parts.

A global behaviour that outlasts any of its components is a defining characteristic of complex systems (Johnson 2002, 82). The order is not defined by a statistical distribution of each part but the result of self-organizing parts. Elements influence each other and their configurations feedback to the configuration of other elements.

3.2 Basic categories

Our assumption can be backed up with empirical research in cognitive psychology. Eleanor Rosch and her colleagues have shown that there are basic levels in cognitive categories (Rosch and Lloyd 1978). When people are asked, "What are you sitting on?" they prefer to say "chair" rather than a concept of a lower abstraction level like "kitchen chair" or of a higher level such as "furniture".

Basic categories – so the theory Rosch & colleagues had worked out – are characterized by relatively homogeneous sensory-motor affordances. A chair can be associated with bending of one's knees, but the lower abstraction level like kitchen chair does not add anything important to the basic level of our (body) involvement with reality. And the higher abstraction level is too abstract to imagine (e.g. visualize) some concrete interaction with a real object.

First line of illustration – common sense example car: no. 8

We do have interactions with vehicles but the concept is too broad to imagine specific kinds of interactions. These necessary interactions for riding a bike or drive a car are so different that we cannot subordinate all of them in an effective cognitive representation. There is no visual presentation, no gestalt of a self-contained whole for "vehicle".

On the other hand: To drive needs certain cognitive and motoric operations. They are all pretty well covered by the general concept of a car. The differences between different passenger cars of different brands do not add any significant feature of movement or interaction. Even the different cognitive and motoric challenges of a passenger car and a truck are of minor importance for our internal presentation of a car.

Second line of illustration – specialist knowledge example teaching: no. 8

What is the "basic level" of teaching? Looking at Figure 4 we certainly can dismiss the two highest levels of abstraction: Politics and institution. Writing curricula is maybe also a certain candidate for exclusion as specialists mainly do this work or – even when teacher design their own curriculum – they do this not regularly on a daily basis.

If we investigate the bottom of the hierarchy we mentioned already that micro interventions (level of educational interactions) is not suitable for educational design. So there remain the levels of educational scenarios, educational ensemble and modules. Educational scenarios are in educational textbooks listed under the concept of teaching methods. In our taxonomy we understand as methods for example group work, fish bowl, brainstorming but we also include methods to implement a certain variety of experiential learning like visit, excursion, exploration, hands-on training, internship, project and legitimate peripheral participation. But we do not include experiential learning or teacher-centered teaching, problem-based learning, and learning through case studies. These concepts do not have a direct and concrete cognitive and corporeal involvement of teachers and students. They are not basis categories of teaching.

Similar considerations apply for the levels of educational ensembles and modules. In educational ensembles teacher put together their different teaching methods they have used to reach a predefined teaching goal in a certain subject matter area. And modules are the European parlance for the level of assessment, where student performance has to be graded.

So it turned out that there are three levels of abstractions where teacher and students are involved cognitively and bodily: scenarios, ensembles and modules. All three of them could work as basic teaching level. So why not group them together? We claim that this detailed break up of reality into three abstraction levels is necessary as these levels are liable to different laws and educational principles. It easier and more convenient educate apprentice-teacher when we address each of these categories separately. It could be argued that one big problem of the educational sciences lies in the fact that these different layers of abstractions are not observed accordingly.

Taken our example of lesson planning it is obvious that educational ensembles or even modules are too big, because they do not fit in the still (at least in our countries Austria and Germany) predominant 40-50 minutes time frame of teaching units. This is the reason why there are so many German books on teaching methods und lesson planning. At the same time we are lacking books on content blocks (Educational Ensembles in our parlance) and assessments of learning (Modules in EU parlance). In the follow up of the PISA-studies – where the German speaking countries have bad rankings – one line of discussion criticizes these small teaching units, as they require a relatively low level of planning. It is argued that German-speaking countries need to revise the structure of their educational framework similar as the PISA high ranked countries like Finland had demonstrated (Sahlberg 2011).

3.3 Graded categories, prototypes and family resemblance

In other experiments it turned out that people do have privileged concepts in their mind, which they connect strongly to a certain level of abstraction. For instance if probands were presented objects like chair or telephone and had to decide how good these items could figure as examples for furniture they came up with a ranking where chair, sofa, couch and table are the top ranked items and sewing machine, stove, refrigerator and telephone are those item least connected to the abstraction "furniture" (Rosch 1975).

These and similar results led to prototype theory: Instead of clear-cut categories in the Aristotelian sense where all items of a certain category share some of their properties Rosch and others claimed that there are graded categories where some objects are more central to the this category than others.

The cognitive linguist George Lakoff (1987) added the idea of family resemblance as another supporting evidence of prototype theory. The concept of family resemblance was developed by Ludwig Wittgenstein (1961) and claims that members of a specified category are not simply related by sharing similar features but linked by a chain of intermediate members with whom they do share some features. So it could happen that one family member of a certain category, situated far away from the prototype, do not even share one single property with another member of this category, which also can be conceptualized with a long distance to the central example (=prototype).

The primary common sense example given in this respect is "bird". In the Aristotelian understanding a bird may be defined with common properties like feathers, a beak and the ability to fly. In contrast, prototype theory would consider a category like bird as consisting of different elements which have unequal status – e.g. a robin is more prototypical of a bird than, say a penguin.

First line of illustration – common sense example car: no. 9

There are many different cars but when we try to visualize one the chance that we imagine a normal passenger car is the highest. SUVs, pickups or trucks are not typical examples; they do not form the prototype of our concept of "car". Cleary this could change with future development and usage behaviour and there are also some cultural differences to keep in mind.

Second line of illustration – specialist knowledge example teaching: no. 9

It is much more difficult to visualise a typical method for experiential learning. One reason for this difficulty is that teaching methods are no objects that one can touch, grasp or shape visually. Another reason is that in our daily teaching practice – at least in the German speaking countries – experiential learning approaches are still not very common. Studies show that about two-thirds of teaching interaction is still done in the teacher-centred presentation mode (Seifried and Klüber 2006, 8) We still visualize a typical learning situation with the teacher in the front of the classroom talking only supported from time to time by some media (blackboard, computer and beamer, interactive whiteboard etc.).

If we would plan an experiment and teacher present different methods of experiential learning like visit, excursion, exploration, hands-on training, internship, project, legitimate peripheral participation we would perhaps get a sound ranking. We assume that the methods of "hands-on training" or "internship" ranks higher (e.g. is more prototypical for experiential learning approaches) than visit or excursion. But it could also be that "project" would get the highest rank as this method is well known and can be integrated into "standard" teaching relatively easy. We are not sure as we are lacking empirical data.

To look for features, which are common in multiple instances, there is a tendency to seek for a common denominator. But we have learned from prototype theory and from the concept of family resemblance that this kind of procedure does not cover reality. As a result rich concepts and their many variations are reduced to a common denominator. It would be an attempt to find "unity in multiplicity" as Bortoft (1996) referring to Goethe adeptly called this kind of abstraction strategy.

The members of a category are not simply related by sharing identical or similar features but linked by a chain of intermediate members with whom they do share (some) features. Two instances (family members) are therefore related to each other, but not by abstract commonalities, rather by being unfolded from the same primal phenomenon (so to speak from the same "ancestor" if we stick with the metaphor of "family resemblance"). – By the way: we do not force this relationship between Goethe and Wittgenstein. According to Buchholz (2006) Wittgenstein refers to Goethe's morphology when he introduces the concept of family resemblance.

4 Summary: Grasping the wholeness, the gestalt

Taken this argumentation into account we have to distinguish between two different kinds of "right" levels. One type is just motivated by a certain practical problem under a given set of inclusive hierarchical strata. Here we ask: "What is the appropriate abstraction stratum to solve the problem in question?" The other type of "rightness" is stimulated by a consideration on the system level: "Do we have the appropriate structure and framework to solve the posed problem?" This distinction could be boiled down to the famous dictum by Peter Drucker (1963): "doing the right things and doing things right". "Doing the right things" is committed to an overall perspective and is therefore more important than "doing things right". But not always are we in the position to design the architecture of the whole system and then we have to limit our efforts for the solution of a problem under preset conditions.

The important consequence for our argumentation for choosing the "right" level under a given framework is the following conclusion: The "right" level of abstraction is one that is linked with basic human categories *and* is rich enough to consist of a manageable amount of family members which assures variety not only in details but also in important and characteristic traits. Our suggestion is that at least 20-25 family members could be a good starting point and Dunbar's number – which lies "between 100 and 230, with a commonly used value of 150" ("Dunbar's Number" 2013) an appropriate endpoint. Dunbar's number is a suggested cognitive limit to the number of people with whom one can maintain stable social relationships. This seems also a good approximation to the metaphor of family resemblance: Is the family bigger than Dunbar' number she is not manageable anymore, the family ties weaken and the community stops to function as a family. More than 150 family members in the chosen level of abstraction might already go too much into details for writing useful and practical patterns for a certain domain level.

When we write patterns we always should consider the impacts for the higher and lower hierarchical level of abstractions: Keeping in mind this triad of hierarchical levels means that our focus is always on the (relatively seen) medium levels. We will always see members of that category, which show both similarity and variation. The similarity is due to the fact that members of that category have taken the same paths in their development e.g. in their history of unfolding. The variation in contrast is due to the fact that the process of unfolding on that level has not ended and therefore the members still unfold in different ways.

The term "unfold" signifies that the possibilities of the future development are already there but has still not evolved. It depends of different effects if these hidden properties have a chance to develop and it what direction they may evolve or mature. **Writing patterns mean to catch the wholeness already intrinsically present**. To grasp this essence, the whole form we need to look into the past (lower level of abstraction) and the prospective future (higher level of abstraction). Only then we will have a chance to start to grasp the whole form – the gestalt. The whole is already present as a nucleus in each concrete instance. More and more wholeness is gained in the process of unfolding.

More concrete representations capture more of the wholeness ("Gestalthaftigkeit"). A very concrete representation shows a fully unfolded instance that is no longer open to differentiations in the potential designs ("Gestaltbarkeit" is missing) (Kohls 2009). It is therefore, that **patterns are abstractions on a medium-level e.g. have to be written for a (relatively seen) medium level in the hierarchy of abstractions we are interested in**. They are concrete enough to let us see the whole (the "gestalt") while at the same time they are not finite instantiations. While there are abstract representations used to communicate and capture patterns, the actual pattern includes all the forms possible. **A pattern constrains the forms but it does not fully define them. It is a form category and not just an abstract form.**

Acknowledgements

Many thanks to our shepherd Antonio Maña. We gave him a hard time as this paper is not an easy read (even the authors still struggle to understand some of the parts…). It is a journey, and the content is still changing…therefore the patience and guidance of Antonio is much appreciated.

Bibliography

Alexander, Christopher. 1979. *The Timeless Way of Building*. Oxford University Press.

Baumgartner, Peter. 2006. “Unterrichtsmethoden Als Handlungsmuster-Vorarbeiten Zu Einer Didaktischen Taxonomie Für ELearning.” In *DeLFI*, 4:51–62. http://peter-baumgartner.at/schriften/article-de/handlungsmuster-taxonomiepdf.pdf.

———. 2011. *Taxonomie von Unterrichtsmethoden: Ein Plädoyer Für Didaktische Vielfalt*. Münster Westf: Waxmann.

Baumgartner, Peter, and Sabine Payr. 1997. “Methods and Practice of Software Evaluation. The Case of the European Academic Software Award.” In *Proceedings of ED-MEDIA 97–World Conference on Educational Multimedia and Hypermedia*. http://medida.bildungstechnologie.org/mdd_2005/mdd_2001/easa-evaluation.pdf.

Becker, Gerold, Andreas Feindt, Hilbert Meyer, Martin Rothland, Lutz Stäudel, and Ewald Terhart, ed. 2007. *Guter Unterricht. Maßstäbe Und Merkmale - Wege Und Werkzeuge.* Vol. Friedrich Jahresheft XXV. Seelze: Friedrich Verlag.

Bjørner, Dines. 2005. *Software Engineering 1: Abstraction and Modelling: V. 1*. 2006th ed. Springer.

Bobik, Joseph. 1963. “Pattern Recognition Mechanisms and St. Thomas’ Theory of Abstraction.” *phlou Revue Philosophique de Louvain* 61 (69): 24–43.

Bohm, David. 1981. *Wholeness and the Implicate Order*. London; Boston: Routledge & Kegan Paul.

Böhmann, Marc, and Thomas Klaffke. 2010. “Die Neuen Kommen! Gut Starten in Schule Und Kollegium. Supplement Zum Friedrich Jahresheft.” In *Friedrich Jahresheft*. Seelze: Friedrich Verlag.

Bortoft, Henri. 1996. *The Wholeness of Nature : Goethe’s Way of Science*. Edinburgh: Floris Books.

Buchholz, Kai. 2006. *Ludwig Wittgenstein*. Frankfurt; New York: Campus.

Buschmann, Frank. 2007. *Pattern-oriented Software Architecture, Vol. 5, Vol. 5,*. Chichester, England; Hoboken, N.J.: Wiley. http://www.myilibrary.com?id=85603.

Coplien, James O. 1996. *Software Patterns*. New York; London: SIGS.

Drucker, Peter F. 1963. *Managing for Business Effectiveness*. Boston, Ma.: Harvard Business Review Reprint Service.

“Dunbar’s Number.” 2013. *Wikipedia, the Free Encyclopedia*. https://en.wikipedia.org/w/index.php?title=Dunbar%27s_number&oldid=566409456.

Erdmann, Benno. 1892. *Logische Elementarlehre*. Max Niemeyer.

Feyerabend, Paul. 1999. *Conquest of Abundance: a Tale of Abstraction Versus the Richness of Being*. Chicago: University of Chicago Press.

Fine, Kit. 2002. *The Limits of Abstraction*. Oxford; New York: Oxford University Press.

Flechsig, Karl-Heinz. 1983. *Der Göttinger Katalog Didaktischer Modelle : Theoretische Und Methodologische Grundlagen*. Göttingen ;Nörten-Hardenberg: Zentrum f. didakt. Studien.

———. 1996. *Kleines Handbuch Didaktischer Modelle*. Eichenzell: Neuland Verl. für Lebendiges Lernen.

Gabriel, Richard P. 1996. *Patterns of Software: Tales from the Software Community*. Oxford University Press Inc, USA.

Gleick, James. 2011. *The Information: a History, a Theory, a Flood*. New York: Pantheon Books.

Goodman, Nelson. 1976. *Languages of Art: An Approach to a Theory of Symbols*. Indianapolis: Hackett.

Hartmann, Nicolai. 1964. *Der Aufbau Der Realen Welt. Grundriß Der Allgemeinen Kategorienlehre*. 3rd ed. Gruyter.

Heidegger, Martin. 2008. *Being and Time*.

Hofstadter, Douglas R. 1979. *Gödel, Escher, Bach: An Eternal Golden Braid*. New York: Basic Books.

Holland, John. 2000. *Emergence : from Chaos to Order.* Oxford: Oxford University Press.

Johnson, Steven. 2002. *Emergence : the Connected Lives of Ants, Brains, Cities, and Software*. 1st Touchstone ed. New York: Touchstone.

Johnson-Laird, Philip Nicholas. 1983. *Mental Models: Towards a Cognitive Science of Language, Inference, and Consciousness*. Harvard University Press.

Kaschek, Roland. 2004. “A Little Theory of Abstraction.” In *Modellierung*, 75–92. http://cs.emis.de/LNI/Proceedings/Proceedings45/GI-Proceedings.45-6.pdf.

Kohls, Christian. 2009. “E-Learning Patterns - Nutzen und Hürden des Entwurfsmuster-Ansatzes.” In *E-Learning 2009: Lernen im digitalen Zeitalter*, edited by Nicolas Apostolopoulos, Hoffmann Harriet, Veronika Masmann, and Andreas Schwill, 61–72. Münster: Waxmann Verlag.

Lakoff, George. 1987. *Women, Fire, and Dangerous Things: What Categories Reveal About the Mind*. Chicago: University of Chicago Press.

Lave, Jean, and Etienne Wenger. 1991. *Situated Learning: Legitimate Peripheral Participation*. Cambridge University Press.

Lea, Doug. 1994. “Christopher Alexander: An Introduction for Object-oriented Designers.” *SIGSOFT Softw. Eng. Notes* 19 (1) (January): 39–46. doi:10.1145/181610.181617. http://doi.acm.org/10.1145/181610.181617.

MacLeod, Mary C., and Eric M. Rubenstein. 2005. “Universals.” *Internet Encyclopedia of Philosophy*. http://www.iep.utm.edu/universa/.

Mitchell, Melanie. 2009. *Complexity: a Guided Tour*. Oxford [England]; New York: Oxford University Press.

Polanyi, Michael. 1969. *Knowing and Being: Essays by Michael Polanyi*. Univ of Chicago Pr.

———. 1974. *Personal Knowledge: Towards a Post-critical Philosophy*. Corr. Ed. University of Chicago Pr.

———. 2009. *The Tacit Dimension*. Reissue. University of Chicago Press.

Rising, Linda. 2007. “Understanding the Power of Abstraction in Patterns.” *IEEE Software* 24 (4) (August): 46–51.

Rosch, Eleanor. 1975. “Cognitive Representations of Semantic Categories.” *Journal of Experimental Psychology: General* 104 (3): 192–233. doi:10.1037/0096-3445.104.3.192.

Rosch, Eleanor, and Barbara L. Lloyd. 1978. *Cognition and Categorization*. John Wiley & Sons Inc.

Sahlberg, Pasi. 2011. *Finnish Lessons: What Can the World Learn from Educational Change in Finland?* New York: Teachers College Press.

Seifried, Jürgen, and Christina Klüber. 2006. *Unterrichtserleben in Schüler- Und Lehrerzentrierten Unterrichtsphasen*. Konstanz :: Bibliothek der Universität Konstanz,.

Weisberg, Michael. 2013. *Simulation and Similarity: Using Models to Understand the World*. Oxford Univ Pr.

Wittgenstein, Ludwig. 1961. *Philosophical investigations*. New York: Macmillan.

Wundt, Wilhelm. 1907. *Logik der Exakten Wissenschaften*. Stuttgart: F. Enke.